

MATH-324 Abstract Algebra

Credit hours: 3-0

Prerequisite: None

Course Objectives: This course aims to introduce students to the basic concepts of group theory. Algebra, developed by the Muslims, started as generalized arithmetic but went on to deal with linear, quadratic, cubic and quartic equations and systems of such equations. Later, Galois and Abel developed group theory to prove that there is no canonical solution of higher order polynomial equations by means of radicals. Two further branches of development followed the solution of simultaneous linear equations (leading to linear algebra) and the more formal structure of groups and their extensions (leading to rings and fields). This course presents the basic concepts of group theory and an introduction to ring theory.

Detailed Course Contents: Sets and relations, partitions and equivalence relations, binary operations, group, dihedral group, quaternion group, group of n^{th} roots of unity, group of residues, general linear group, subgroups, order of a group, order of an element of a group, cyclic groups, cyclic subgroups, generating sets, permutation, group of permutations, cycles, transpositions, even and odd permutations, alternating groups, decomposition of a permutation into disjoint cycles, cosets, index of a subgroup in a group, Lagrange's theorem, consequences of Lagrange's theorem, normal subgroups, product of subgroups, group homomorphism, properties of a homomorphism, kernel of a homomorphism, isomorphism, factor groups, the first isomorphism theorem, center subgroup, rings, subrings, integral domains, ideals.

Learning Outcomes: On successful completion of this course, students will know about

- Equivalence relations on sets, equivalence classes.
- Binary operation, group, subgroup, order of a group
- Dihedral group, quaternion group, group of n^{th} roots of unity, group of residues, general linear group.
- Permutation, group of permutations, cycles, transpositions, even

permutations, odd permutations.

- Alternating group, decomposition of a permutation into disjoint cycles.
- Coset, index of a subgroup in a group, Lagrange's theorem.
- Normal subgroups, center subgroup, commutator subgroup.
- Group homomorphism, kernel of a homomorphism, isomorphism.
- Factor group, the first isomorphism theorem.
- Product of subgroups.
- Rings, subrings, integral domains, ideals.

Textbook: J. A. Gallian, Contemporary Abstract Algebra, 8th ed. Brooks/Cole, CA, 2013.

Reference book:

1. W. Keith Nicholson, Introduction to Abstract Algebra, (3rd edition), 2007, John Wiley & Sons.
2. J. H. Fraleigh: A first course in abstract algebra (7th edition), 1998, Addison-Wesley publishing.
3. N. Herstein, Abstract Algebra, third edition, 1995, Prentice Hall.

Weekly Breakdown		
Week	Section	Topics
1	Ch. 0	Equivalence relations, partition, equivalence classes partition, examples and related results. Functions, types of functions and composition of functions.
2	Ch. 1,2	Binary operations, groups, definitions and examples. Group of integers modulo n, dihedral group.
3	Ch. 2	Group of quaternions, group of n^{th} roots of unity, general linear group. Elementary properties of groups.
4	Ch. 3	Subgroups, order of a group, order of an element of a group, subgroup tests, cyclic subgroups, subgroup generated by a subset of a group. The center subgroup, centralizer and normalizer subgroups.

5	Ch. 4	Cyclic groups, Properties of the cyclic groups, related theorems, examples, generators of finite cyclic groups, generators of Z_n .
6	Ch. 5	Permutation of a set, permutation group of a set, symmetric group S_n , cycle notation, product of disjoint cycles, product of disjoint cycles commute.
7	Ch. 5	Order of a permutation, transpositions (2-cycles) and related theorems, even and odd permutations, the alternating group of degree n .
8	Ch. 7	Cosets, properties of cosets, index of a subgroup in a group, definition examples and related results, and the theorem of Lagrange.
9	Mid Semester Exam	
10	Ch. 7	Consequences of Lagrange's theorem, converse of the Lagrange theorem, Product of subgroups.
11	Ch. 9	Definition examples of direct product of groups. Normal subgroups, definitions, examples and related results.
12	Ch. 9	Factor groups, examples and related theorems.
13	Ch. 10	Group homomorphism, Properties of a homomorphism, kernel of a homomorphism, definitions, examples and related results.
14	Ch. 10	Properties of elements under homomorphism, properties of subgroups under homomorphism, kernels are normal subgroups. First isomorphism theorem and its applications.
15	Ch. 12	Rings, definition and examples, properties of rings, uniqueness of identity and inverses.
16	Ch. 12	Subrings, definition and examples, subring test, applications of subring tests. The group of unit elements of a commutative ring.
17	Ch. 13, 14	Boolean ring, Integral domains, ideals, ideal test.
18	End Semester Exam	